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RESONANCE BENDING WAVES IN A CYLINDRICAL SHELL UNDER A MOVING RADIAL LOAD

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Analysis of axisymmetric wave processes in infinite cylindrical systems shows [1, 2] that critical velocities of motion exist in the axial direction of the surface load that forms resonance perturbations. If the load velocity agrees with the "rod" velocity ($c_s = \sqrt{E/\rho}$), longwave resonance of the longitudinal vibrations is realized. Another critical velocity corresponds to the medium-wave part of the spectrum and to the minimum of the dispersion curve of the first mode.

The asymptotic of resonance wave growth in shells is obtained in [1-3] for comparatively large values of the time ($t \rightarrow \infty$). Applicability of the asymptotic solution for finite values of the time is investigated only for low-frequency longitudinal resonance processes [2, 4, 5]. A bending resonance wave asymptotic is obtained below for a different kind of load and its applicability is clarified for quantitative estimates in systems of bounded length. The kind of load is determined for which the perturbations grow substantially more rapidly than in other cases.

Formulation of the Problem

Shell dynamics is described by the linear equations of classical Kirchhoff-Love theory:

$$\ddot{u} = u''_x + \nu w'_x, \quad \ddot{w} = -\nu u'_x - w - \varepsilon u_x^{IV} + Q/h, \quad \varepsilon = h^2/12, \quad (1)$$

where u and w are the shell displacements in the axial x and radial directions; h is the shell thickness, and Q is the acting load. Taken as units of measurement are $c = \sqrt{E/[\rho(1-\nu^2)]}$ the speed of sound in a thin plane (E is Young's modulus, ν is the Poisson ratio), R is the

shell radius, and ρ is its density. The initial conditions are zero. Symmetry conditions ($w_x' = w_x''' = 0, u = 0$) are satisfied in the $x = 0$ plane.

Loads of two kinds are examined: a) over the shell surface along its axis a step normal pressure wave $Q = H_0(t)H_0(c_0t - |x|)$ moves at the velocity c_0 [$H_0(z)$ is the Heaviside function]; b) the pressure amplitude in the wave varies according to a sinusoidal law $Q = H_0(t) \cdot H_0(c_0t - |x|) \sin(q_0|x| - \omega_0t)$ ($\omega_0 = q_0c_0$, q_0 is the mode frequency). The situation when a local load oscillating at the frequency ω_0 : $Q = H_0(t)H_1(x) \sin \omega_0t$ [$H_1(z)$ is the Dirac function] is applied in the section $x = 0$ is investigated as a particular case.

Applying the Laplace integral transform in t (symbol L , parameter p) and the Fourier integral transform in x (symbol F , parameter q) to (1), we obtain the solution in LF transforms:

$$\begin{aligned} w^{LF} &= \frac{Q^{LF}}{h} \frac{p^2 + q^2}{A(p, q)}, \quad u^{LF} = -\frac{Q^{LF}}{h} \frac{ivq}{A(p, q)}, \\ A(p, q) &= (p^2 + q^2)(1 + \varepsilon q^4 + p^2) - v^2 q^2 \end{aligned} \quad (2)$$

[$A(p, q)$ is the dispersion operator of the system].

The equation $A(p = iqc, q) = 0$ describes the dependence of the phase velocity c on the wave number q ($q = 2\pi/\lambda$, λ is the wavelength). In particular, it has two roots for shells

$$\begin{aligned} c_1(q) &= \sqrt{a - b}, \quad c_2(q) = \sqrt{a + b} \\ (a &= (1/2)(\varepsilon q^2 + q^{-2} + 1), \quad b = (1/2)\sqrt{(\varepsilon q^2 + q^{-2} - 1)^2 + 4v^2 q^{-2}}). \end{aligned} \quad (3)$$

As is shown in [1], singular points on the phase curves in the q, c plane in which the phase and group ($\bar{C} = c + qdc/dq$) velocities are equal determine the critical velocities of load motion that form resonance perturbations. The singular point of the first mode $q_* \neq 0, dc/dq|_{q=q_*} = 0$ is investigated below, where the phase velocity reaches the absolute minimum c_* .

For $v = 0$, the dispersion operator degenerates into the product of two independent operators corresponding to bending waves in a rod on an elastic base (with unit stiffness of the bedding coefficient) and longitudinal waves in the rod. We are interested in the first mode from which $q_* = (12/h^2)^{1/4}$ and $c_* = (h^2/3)^{1/4}$ follow. There are no analogous expressions in explicit form for shells; consequently, by expanding (3) in powers of h we find the approximate expressions

$$q_* \approx [12(1 - v^2)/h^2]^{1/4}, \quad c_* \approx [h^2(1 - v^2)/3]^{1/4}.$$

The originals are not determined successfully in explicit form from the solution of (2). We shall seek the perturbation asymptotic in the system for large times from the beginning of load action ($t \rightarrow \infty$) by using the method of inverting double integral transforms in the neighborhood of the ray $x = c_*t + \eta$, proposed in [2]. The action is symmetrical relative to the plane $x = 0$; consequently, we henceforth examine the domain $x \geq 0$.

Traveling Step Load

For $c_0 = c_*$, $q = q_*$, we obtain the following asymptotic solution ($t \rightarrow \infty$):

$$\begin{aligned} w(x, t) &\sim -\frac{1 - c_*^2}{\pi h q_*^2 c_* (\bar{c}_2^2 - c_*^2)} \left(\frac{t}{\varphi}\right)^{1/2} (F_1(\kappa) \cos \eta q_* + F_2(\kappa) \sin \eta q_*), \\ u(x, t) &\sim -\frac{v}{\pi h q_*^3 c_* (\bar{c}_2^2 - c_*^2)} \left(\frac{t}{\varphi}\right)^{1/2} (F_2(\kappa) \cos \eta q_* - F_1(\kappa) \sin \eta q_*), \\ \bar{c}_2 &= c_2(q_*), \quad \varphi = \frac{1}{2} \frac{d\bar{C}_1}{dq} \Big|_{q=q_*}, \quad \kappa = -\eta(\varphi t)^{-1/2}, \quad \eta = x - c_*t, \\ F_1(\kappa) &= \int_0^\infty \frac{\cos \kappa y \sin y^2}{y^2} dy = \sqrt{\frac{\pi}{2}} \left(\cos\left(\frac{\kappa^2}{4}\right) + \sin\left(\frac{\kappa^2}{4}\right) \right) - \frac{\pi|\kappa|}{2} \left[C\left(\frac{\kappa^2}{4}\right) - S\left(\frac{\kappa^2}{4}\right) \right], \end{aligned}$$

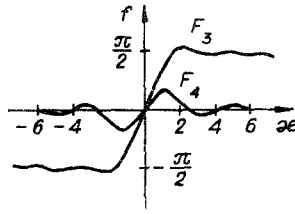


Fig. 1

$$F_2(\kappa) = \int_0^{\infty} \frac{\cos \kappa y (1 - \cos y^2)}{y^2} dy = \sqrt{\frac{\pi}{2}} \left(\cos \left(\frac{\kappa^2}{4} \right) - \sin \left(\frac{\kappa^2}{4} \right) \right) - \frac{\pi |\kappa|}{2} \left[1 - C \left(\frac{\kappa^2}{4} \right) - S \left(\frac{\kappa^2}{4} \right) \right] \quad (4)$$

(graphs of the functions F_1 and F_2 are presented in [2], S and C are Fresnel integrals). If we speak about a rod on an elastic basis, then c_2 must be replaced by 1 and v by 0 in (4) and further.

The asymptotic (4) agrees with that obtained in [2] to the accuracy of the misprints admitted there. It is seen that as the step load moves at the critical velocity c_* a resonance process is formed in the system: the amplitude of the quasistationary envelope of the bending perturbations grows without limits as $t^{1/2}$ as time elapses, the domain occupied by the perturbations expands also as $t^{1/2}$, and the bending-wave carrier frequency equals q_* .

Traveling Oscillating Load

For $c_0 = c_*$ and $q_0 = q_*$, the asymptotic ($t \rightarrow \infty$) of the solution is found in the form

$$\begin{aligned} w(x, t) &\sim -\frac{1 - c_*^2}{2\pi h q_* c_* (\bar{c}_2^2 - c_*^2)} \left\{ t \left[\cos \eta q_* \left(\frac{\pi}{2} \left[1 + 2 \left(\frac{x}{c_* t} - 1 \right) H_0(-\eta) \right] + \right. \right. \right. \\ &+ F_3(\kappa) \left. \left. \left. - F_4(\kappa) \sin \eta q_* \right] + \frac{1}{q_*} \left(\frac{t}{\varphi} \right)^{1/2} (F_2(\kappa) \cos \eta q_* + F_1(\kappa) \sin \eta q_*) \right\}, \\ u(x, t) &\sim -\frac{v}{2\pi h q_*^2 c_* (\bar{c}_2^2 - c_*^2)} \left\{ t \left[-\sin \eta q_* \left(\frac{\pi}{2} \left[1 + 2 \left(\frac{x}{c_* t} - 1 \right) H_0(-\eta) \right] + \right. \right. \right. \\ &+ F_3(\kappa) \left. \left. \left. + F_4(\kappa) \cos \eta q_* \right] - \frac{1}{q_*} \left(\frac{t}{\varphi} \right)^{1/2} (F_1(\kappa) \cos \eta q_* - F_2(\kappa) \sin \eta q_*) \right\}, \\ F_3(\kappa) &= \int_0^{\infty} \frac{\sin \kappa y \sin y^2}{y^3} dy, \quad F_4(\kappa) = \int_0^{\infty} \frac{\sin \kappa y (1 - \cos y^2)}{y^3} dy. \end{aligned} \quad (5)$$

The integral $F_3(\kappa)$ tends for large values of $|\kappa|$ to $\pm\pi/2$, while $F_4(\kappa)$ tends to zero (Fig. 1).

Therefore, under the action of a sinusoidal load moving at the critical velocity c_* and oscillating at the frequency q_* , the amplitude of the bending perturbations envelope grows in proportion to t . This is more rapid than for the step load, and the growth can be explained by the presence in the load itself of the frequency q_* of the form that corresponds to the resonance mode of the vibrations.

If $F_3(\kappa)$ is approximately replaced asymptotically by the equivalent function

$$f(\kappa) = \kappa H_0(\pi/2 - |\kappa|) + (\pi/2) H_0(|\kappa| - \pi/2) \text{sign } \kappa$$

and the contribution of the functions F_1 , F_2 , and F_4 , which are asymptotically not essential as compared to $F_3(\kappa)$, is not taken into account, then we obtain an approximate estimate of the solution

$$\begin{aligned} w(x, t) &\sim -\frac{t(1 - c_*^2) \cos \eta q_*}{2\pi h q_* c_* (\bar{c}_2^2 - c_*^2)} \left\{ \left(\kappa - \frac{\pi}{2} \text{sign } \eta \right) H_0 \left(\frac{\pi}{2} - |\kappa| \right) + \frac{\pi x}{c_* t} H_0(-\eta) \right\}, \\ u(x, t) &\sim -\frac{v \sin \eta q_*}{q_* (1 - c_*^2) \cos \eta q_*} w(x, t). \end{aligned} \quad (6)$$

Hence it is seen that at a fixed point x the amplitude of the envelope of an oscillogram is proportional to x .

Local Oscillating Load

Let us consider the dependence $\omega = \omega(q)$. Two branches exist in the ω, q plane: $\omega_{1,2} = qc_{1,2}$. The point $\omega = 1$ of the curve $\omega_2(q)$ is singular since here $d\omega/dq = 0$ [6]. Solutions valid for $\omega_0 < \omega$ are represented below, where ω constrains the perturbation spectrum $(0, \bar{\omega})$, acceptably being described by classical theory [2].

For $\omega_0 > 1$ the asymptotic obtained ($t \rightarrow \infty$) has the form

$$\begin{aligned}
 w(x, t) &= w_1(x, t) + w_2(x, t), \quad u(x, t) = u_1(x, t) + u_2(x, t), \\
 w_{1,2}(x, t) &\sim -(1/2)W_{1,2}\{P_1(\kappa_{1,2}) \cos(\omega_0 t - q_{1,2}x) - \\
 &\quad - P_2(\kappa_{1,2}) \sin(\omega_0 t - q_{1,2}x)\}, \\
 u_{1,2}(x, t) &\sim -(1/2)U_{1,2}\{P_1(\kappa_{1,2}) \sin(\omega_0 t - q_{1,2}x) + \\
 &\quad + P_2(\kappa_{1,2}) \cos(\omega_0 t - q_{1,2}x)\}, \\
 P_1(\kappa) &= 1 + \left[C\left(\frac{\kappa^2}{4}\right) + S\left(\frac{\kappa^2}{4}\right) \right] \text{sign } \kappa, \quad P_2(\kappa) = \left[C\left(\frac{\kappa^2}{4}\right) - S\left(\frac{\kappa^2}{4}\right) \right] \text{sign } \kappa, \\
 W_{1,2} &= \frac{1 - c_{1,2}^2(q_{1,2})}{2h\omega_0 \bar{C}_{1,2}(q_{1,2}) [c_{2,1}^2(q_{1,2}) - c_{1,2}^2(q_{1,2})]}, \\
 U_{1,2} &= \frac{v}{2h\omega_0 q_{1,2} \bar{C}_{1,2}(q_{1,2}) [c_{2,1}^2(q_{1,2}) - c_{1,2}^2(q_{1,2})]}, \\
 \bar{C}_{1,2} &= \frac{d\omega_{1,2}(q)}{dq}, \quad \kappa_{1,2} = (\bar{C}_{1,2}(q_{1,2})t - x)(\varphi_{1,2}t)^{-1/2}, \quad \varphi_{1,2} = \frac{1}{2} \frac{d\bar{C}_{1,2}}{dq} \Big|_{q=q_{1,2}}
 \end{aligned} \tag{7}$$

(q_1 and q_2 are values of the wave numbers determined by points of intersection of the line $\omega = \omega_0$ with the phase curves ω_1 and ω_2). For a rod on an elastic basis $u_1 = u_2 = w_2 = 0$.

Starting with $z \sim 1$, the functions $C(z)$ and $S(z)$ oscillate relative to the mean value 0.5 and approach it as z grows. The maximal deviation from the mean for $z \geq 1$ lies in the 35% limit. Taking this into account, we find the approximate estimate which, as $t \rightarrow \infty$, will be more exact the farther the perturbed domain under consideration stands off from the quasifront:

$$\begin{aligned}
 w_{1,2}(x, t) &\sim -W_{1,2} \cos(\omega_0 t - q_{1,2}x) H_0(\bar{C}_{1,2}(q_{1,2})t - x), \\
 u_{1,2}(x, t) &\sim -U_{1,2} \sin(\omega_0 t - q_{1,2}x) H_0(\bar{C}_{1,2}(q_{1,2})t - x).
 \end{aligned} \tag{8}$$

Expressions (8) have a more accessible form for analysis than does (7). It is seen that the perturbations in the shell consist of two waves: a bending wave is propagated at the velocity C_1 while a longitudinal "shell" wave initiated by bending perturbations moves ahead at the velocity $\bar{C}_2 \gg C_1$. The deflection in the longitudinal wave W_2 (the mean value) is at least an order of magnitude less than W_1 . Hence, there results that the fundamental perturbations in the shell can be computed on the basis of the model of a rod on an elastic basis.

For $\omega_0 = 1$ the asymptotic ($t \rightarrow \infty$) is written as

$$\begin{aligned}
 w(x, t) &\sim -\frac{t^{3/4}}{2\pi h \varphi_0^{1/4}} \{F_5(\kappa_0) \cos t - F_6(\kappa_0) \sin t\} + w_1(x, t), \quad u(x, t) = u_1(x, t), \\
 F_5(\kappa) &= \int_0^\infty \frac{\cos xy \sin y^4}{y^4} dy, \quad F_6(\kappa) = \int_0^\infty \frac{\cos xy (1 - \cos y^4)}{y^4} dy, \\
 \kappa_0 &= x(\varphi_0 t)^{-1/4}, \quad \varphi_0 = \frac{1}{2^4} \frac{d^3 \bar{C}_2}{dq^3} \Big|_{q=0}.
 \end{aligned} \tag{9}$$

Graphs of the integrals $F_5(\kappa)$ and $F_6(\kappa)$ are given in Fig. 2. For the section $x = 0$ the asymptotic (9) is converted to

$$w(0, t) \sim -\frac{t^{3/4}}{6\pi h \varphi_0^{1/4}} \Gamma\left(\frac{1}{4}\right) \sin\left(\frac{3\pi}{4} - t\right) + w_1(0, t), \quad u(0, t) = u_1(0, t) \tag{10}$$

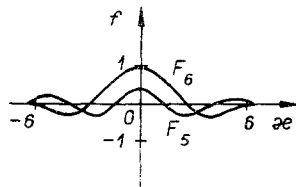


Fig. 2

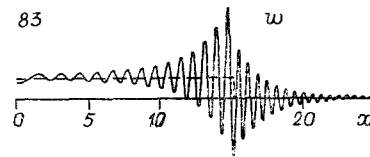


Fig. 3

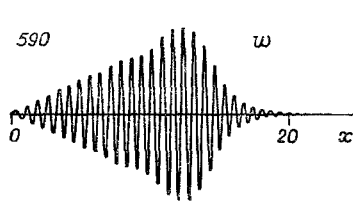


Fig. 4

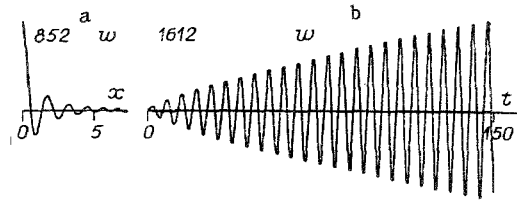


Fig. 5

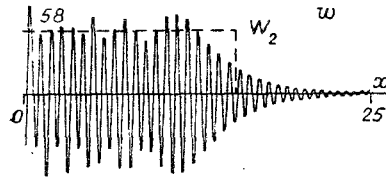


Fig. 6

[$\Gamma(z)$ is the gamma function]. It is seen from (9) and (10) that the perturbation amplitude grows as $t^{3/4}$ in the neighborhood of $x = 0$ under the action of a local oscillating load with $\omega_0 = 1$ and expands as $t^{1/4}$. This result corresponds to the qualitative estimate [6]: resonance perturbations that grow in proportion to $t^{1-1/n}$ with time form in the system under a local strong monochromatic excitation with $\omega = 1$ and propagate along the axis at a decreasing velocity (n is the order of the first derivative $d^n\omega/dq^n$ different from zero). In the case considered, $n = 4$.

For $\omega_0 < 1$, the asymptotic ($t \rightarrow \infty$) for a shell corresponds to (7) and (8), where $w_2 = u_2 = 0$; for a rod on an elastic basis we obtain a solution in which the amplitude does not grow asymptotically with time and tends to zero with distance from the action site.

Numerical Solutions

In order to determine the limits of applicability of the asymptotic solutions found, the initial equations of motion are computed by a finite-difference method according to an explicit scheme. Numerical dispersion is minimized by selecting optimal mesh parameters for which the stability conditions are satisfied and the minimums of the phase velocities of the difference and continual models are closest. As comparison of the phase curves shows, achievement of coincidence of the critical points (q_*, c_*) is possible only for $\tau, \delta \rightarrow 0$ (τ and δ are mesh spacings in the time and the coordinate); consequently, values of q_* and c_* are taken from the difference dispersion relationships in the numerical computations.

Results of numerical computations are represented in Figs. 3-6 for a rod on an elastic basis obtained for $h = 0.05$, $t = 90$, $\tau = \delta = 0.05$ (the maximal amplitude of the deflection is indicated in the upper left corner). According to the computations, the results for shells with the same parameters do not differ in practice from those represented.

A graph of the deflection under the action of a step load traveling at the critical velocity c_* is given in Fig. 3, the dashed line is the static deflection $w = Q/h = 20$. Taking it into account, the difference between the numerical and analytic solutions decreases with time and is already not more than 2% for $t = 120$. It can be considered that the asymptotic (4) is practically exact starting with $t \approx 30$.

A deflection diagram for a traveling sinusoidal load ($q_0 = q_*$, $c_0 = c_*$) is shown in Fig. 4. The resonance perturbation growth rate agrees with the asymptotic estimate (5), the dif-

ference in the maximal values of the amplitude is not more than 3% ($t = 120$). It is seen from an analysis of the perturbation oscillogram obtained numerically that the amplitude behind the front oscillates relative to the mean value $w = q_*x/4h$ determined from (6).

Graphs of the deflection computed for a local oscillating load are represented in Figs. 5 and 6. The numerical results are in good agreement with the analytic results: for $\omega_0 < 1$ the perturbation amplitude does not grow with time; for $\omega_0 = 1$ (Fig. 5a is the diagram for $t = 90$ and b is the oscillogram at the point $x = 0$) the perturbation envelope in the neighborhood of $x = 0$ increases in proportion to $t^{3/4}$; for $\omega_0 > 1$ (Fig. 6, $\omega_0 = \sqrt{2}$) the perturbations propagate with an amplitude oscillating relative to the mean value (dashed line) that is found from (7).

Comparing the numerical and analytic solutions describing bending resonance wave propagation in a cylindrical shell and a rod on an elastic basis shows that the asymptotics obtained determine, with good accuracy, the fundamental perturbations in a system formed after a finite time interval.

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CALCULATION OF STRAINS FOR BRITTLE MATERIALS

TAKING INTO ACCOUNT LIMITING FAILURE

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In order to describe the strain properties of heterogeneous materials there is currently extensive use of a model for an elastic material weakened by a large number of cracks [1-9].

The aim of the present work is to construct a system of fundamental equations for computing the strain properties of brittle materials based on development of a model for a cracked material suggested in [3, 4, 9] taking account of crack growth during deformation.

1. We consider development of an isolated shear crack. Shear crack propagation in a plane arrangement was studied in [6-8] where it was noted that during loading in the end zones of a shear crack separation cracks occur growing in the general case along a curvilinear trajectory.

Experiments [8] show that curvature of a growing separation crack occurs directly adjacent to the end zone of a shear crack. Subsequently, independent of the direction for the plane of a shear crack growth of a separation crack occurs in a plane perpendicular to the direction of least compressive stress.

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